CALCULATING HEAT TRANSFER IN THE REGION OF AN ATTACHED BOUNDARY LAYER

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A method is proposed to calculate heat transfer after a diaphragm in a circular pipe. Satisfactory agreement is obtained between the calculated results and test data obtained with different Re_D and Pr numbers and different ratios of pipe diameter to diaphragm diameter.

Flows with separation and reattachment of the flow in channels have recently gained the attention of a broad range of investigators in connection with the fact that such flows make it possible to significantly intensify convective heat transfer under certain conditions. Several theoretical [1, 2] and experimental [3-5] studies have investigated heat transfer in the attachment region.

It is known that the heat-transfer coefficient in the region of attachment of a boundary layer is maximal and exceeds several times (by factors up to nine) the values seen in the region of fully developed flow [3-5]. It is also interesting to note a very important fact, specifically that a high level of heat transfer is seen not only in the "attached" region but also upstream and downstream of this region. For example, according to data in [4], heat transfer in an attached boundary layer downstream of the attached region is 1.5-2 times greater than in a normal boundary layer developed from the same point. A turbulent boundary layer after attachment of the flow obviously differs considerably from the boundary layer in a normal unperturbed flow.

As in [1, 2], we will adopt a two-layer scheme of turbulent flow in the attached region: the region in which viscosity and the turbulent core exert an effect. In accordance with [1, 2], for $Y_0 \leqslant Y \leqslant Y_1$ in the attached region there exists the relationship

$$(K_1/K_0)_{max} = (Y_1/Y_0)_{max}. \tag{1}$$

For an attached boundary layer in a pipe extending from the attached region to fully developed flow, we adopt a relation similar to Eq. (1):

$$(K_1/K_0) = M(Y_1/Y_0). \tag{2}$$

The value of the coefficient M changes along the longitudinal coordinate X from 1 in the attached region to a certain value M_∞ in the region of flow stabilization.

We approximately assume that the outer boundary of the layer, with the effect of viscosity Y_0 in a circular pipe, corresponds to the dimensionless coordinate $\eta_0 = 40$. In this case, we assume, as in [2], that $(\nu_t)_0/\nu = 16$. With allowance for the relation for $\nu_t = CK^{1/2}Y$, by using Eq. (2) we obtain the following expression for Y_0 in the attached region:

$$Y_0 = \left(\frac{16\nu M^{1/2} Y_1^{1/2}}{CK_1^{1/2}}\right)^{2/3}. \tag{3}$$

Having taken the mean flow velocity over the channel cross section $\bar{u}$ as the characteristic velocity and using (3), we obtain an expression for the complex $\bar{u}Y_0/\nu$:

$$\frac{\bar{u}Y_0}{\nu} = \left(\frac{\sqrt{K_1}}{\bar{u}}\right)^{-2/3}\left(\frac{D}{Y_1}\right)^{-1/3}\left(\frac{\bar{u}D}{\nu}\right)^{1/3}\left(\frac{16M^{1/2}}{C}\right)^{2/3}. \tag{4}$$

The following expression for Nu_D is found from the heat-transfer equation for turbulent flow [1, 2] in the case of a thin attached boundary layer and from Eq. (4):

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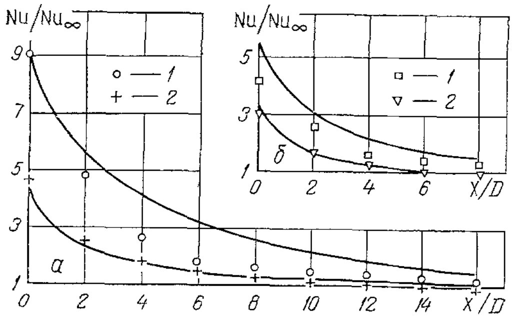

Fig. 1. Comparison of results of calculation of $Nu/Nu_\infty = f(X/D)$ with test data from [3] (water, Pr = 3) for different numbers Re_D and ratios D/d (curves represent calculated results): a: 1) Re_D = 9900, D/d = 4; 2) 10,100 and 1.5; b: 1) Re_D = 102,000, D/d = 3; 2) 131,000 and 1.5.

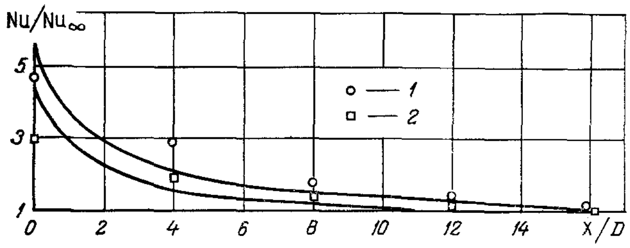

Fig. 2. Comparison of results of calculation of $Nu/Nu_\infty = f(X/D)$ with test data from [3] (water, Pr = 6) D/d = 2; 1) Re_D = 15,800, 2) 100,000 (curves represent calculated results).

$$\mathrm{Nu}_D = \left(\frac{C}{16M^{1/2}}\right)^{2/3}\left(\frac{D}{Y_1}\right)^{1/3}\left(\frac{\sqrt{K_1}}{\bar{u}}\right)^{2/3}\mathrm{Re}_D^{2/3}\left(\frac{t_{\mathrm{axs}}-t_{\mathrm{w}}}{\bar{t}-t_{\mathrm{w}}}\right)\left[\int_0^{Y_1/Y_0}\frac{d\left(\frac{Y}{Y_0}\right)}{\frac{1}{\mathrm{Pr}}+\left(\frac{\nu_{\mathrm{t}}}{\nu}\right)\frac{1}{\mathrm{Pr}_{\mathrm{t}}}}\right]. \tag{5}$$

The expression $(t_{axs} - t_w)/(\bar{t} - t_w)$ for a smooth pipe is approximated by the following formula (see [2]):

$$\frac{t_{\mathrm{axs}}-t_{\mathrm{w}}}{\bar{t}-t_{\mathrm{w}}} = \frac{1.75}{8.5+\mathrm{Pr}} + 1. \tag{6}$$

To calculate heat transfer from (5), it is necessary to know how the quantities M, D/Y_1, and $\sqrt{K_1}/\bar{u}$ change along the pipe from the attached region to completely developed flow.

The value of M changes from 1 in the attached region to the value $M_\infty = (K_1/K_0)_\infty/(Y_1/Y_0)_\infty$ in the region of flow stabilization. We will find the value of M_∞. For the region of stablized flow we take $\eta_0 = 40$. Then

$$(Y_1/Y_0)_\infty = (\eta_1/\eta_0)_\infty = \frac{u_* R}{\nu \cdot 40} = 0.002486\mathrm{Re}_D^{0.875}. \tag{7}$$

Using the data from [6], we found that in the flow stabilization region in a pipe

$$(K_1/K_0)_\infty = 0.1736. \tag{8}$$

With allowance for (7) and (8) we have

$$M_\infty = \frac{69.83}{\mathrm{Re}_D^{0.875}}. \tag{9}$$

We assume a linear change in M along X in the attached region. Then, using (9):

$$M = 1 - \left(1 - \frac{69.83}{\mathrm{Re}_D^{0.875}}\right)\frac{X}{X_n}. \tag{10}$$

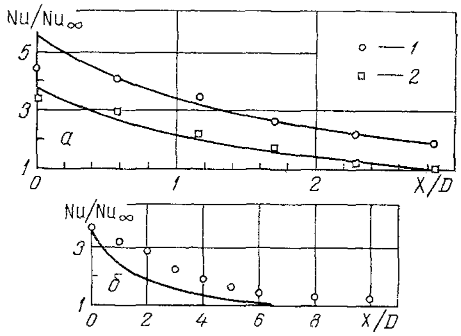

Fig. 3. Comparison of results of calculation of $Nu/Nu_\infty = f(X/D)$ with test data from [5] (air, Pr = 0.7) (curves represent calculated results): a: D/d = 2.33; 1) Re_D = 4178; 2) 47,640; b: D/d = 1.85; Re_D = 48,090.

Let us examine the value of D/Y_1. Its value in the flow stabilization region in the pipe is equal to: $D/Y_1 = D/R = 2$. However, the second boundary value of this quantity in the attached region is indeterminate.

In [1, 2] a constant value equal to about 10 was taken for $(D/Y_1)_{max}$ in the attached region. However, as was shown in [1], such an estimate introduces the most indeterminateness in the calculation of heat transfer.

In this article we attempted to determine $(D/Y_1)_{max}$ by examining test data on heat transfer in a circular pipe. Analysis of the test data in [3, 5] leads us to conclude that heat transfer in the attached region in a circular pipe beyond the diaphragm causing the flow separation can be reliably calculated from the formula

$$(\mathrm{Nu}_D)_{\max} = 0{,}256\mathrm{Re}_D^{2/3}\mathrm{Pr}^{0.4}(D/d)^{2/3}. \tag{11}$$

Comparing Eq. (11) with the analogous relation used in [2] by a semiempirical method, we find a formula to calculate $(D/Y_1)_{max}$ in the attached region:

$$(D/Y_1)_{\max} = \left(\frac{3{,}88\mathrm{Pr}^{0{,}09} + \dfrac{1.14}{\mathrm{Pr}^{1.6}}}{\dfrac{1.75}{8.5+\mathrm{Pr}} + 1}\right)(d/D)^2, \tag{12}$$

Equation (12) is valid when Pr changes from 0.7 to 6 and d/D changes from 2/3 to 1/4. Assuming a linear change in D/Y_1 in the attached region, we obtain the following theoretical formula for (D/Y_1) along the pipe:

$$(D/Y_1) = (D/Y_1)_{\max} - [(D/Y_1)_{\max} - 2]\,\frac{X}{X_n}\,. \tag{13}$$

Local heat transfer can be calculated by Eq. (5) with allowance for (6), (10), (12), and (13) in the case when the length of the region of attached boundary-layer flow X_n is known. There are presently no reliable recommendations for determining this quantity. However, in the case of attachment and subsequent development of a boundary layer in a cylindrical pipe, it can be assumed that the value of this quantity will be close to the value on the initial thermal section [3]. According to the data in [7], the length of the initial thermal section can be determined as a function of Re_D from the formulas

$$(X_n/D) = \frac{4.5\cdot 10^5}{\mathrm{Re}_D} \quad \text{at } \mathrm{Re}_D \leqslant 5\cdot 10^4,$$

$$(X_n/D) = 0.6\mathrm{Re}_D^{1/4} \quad \text{at } \mathrm{Re}_D > 5\cdot 10^4. \tag{14}$$

Data presented in [8] are used to determine the last unknown $(\sqrt{K_1}/\bar{u})$. Since turbulence is high in the attached region, we assume that the dissipation of turbulent energy along the pipe axis from the attached region is the same as in a pipe with swirl vanes at the inlet:

$$\varepsilon_x = 0.8165\,(\sqrt{K_1}/\bar{u})_x = \varepsilon_{\max}\left(\frac{X/D}{\varepsilon_{\max}} + 1\right)^{-n},$$

$$n = 0.113\,(\varepsilon_{\max})^{0.45} \quad \text{at} \quad 35\% < \varepsilon_{\max} < 80\%. \tag{15}$$

Here, ε_{max} is the degree of turbulence beyond the diaphragm in the attached-flow region a distance Y_1 from the wall. On the other hand, according to the data in [2] the turbulent energy in this region can be found from the formula

$$(\sqrt{K_1}/\bar{u})_{max} = 0.25\,(D/d)^2. \tag{16}$$

The value of ε_{max} can be found from Eqs. (15) and (16), and Eq. (15) can be used to calculate $(\sqrt{K_1}/\bar{u})$ along the length of an attached boundary layer.

As in [2], heat transfer is calculated by breaking the integral in Eq. (5) down into three integrals with limits of integration through the thickness of the viscous sublayer, the intermediate region, and the turbulent core.

Results of calculations with Eq. (5) were compared with data obtained in [3, 5] in tests with water and air for different numbers Re_D and different ratios D/d (see Figs. 1-3). The comparsion shows that, on the whole, the calculation agrees with the experiment. The agreement becomes poorer as D/d increases. This can be explained by the fact that the calculation did not take into account the asymmetry of the attached flow after the diaphragm (see [3-5]). Also, Eq. (16) used in the calculations was obtained for a limited range of values of D/d.

NOTATION

D, inside diameter of pipe; d, inside diameter of diaphragm; Re_D, Reynolds number; Pr, Prandtl number; Nu_D, Nusselt number; X, longitudinal coordinate; Y, transverse coordinate; K, critical turbulent energy; u_*, dynamic velocity; ν, kinematic viscosity; $\bar{t}$, mean temperature of fluid in the investigated cross section of the pipe; t_w, temperature of wall; X_n, length of initial thermal section. Indices: max, in the region of the attached boundary layer; 1, on the outer boundary of the turbulent core; 0, on the outer boundary of the layer with the viscosity effect; ∞, in the stabilized region; t, turbulent value.

LITERATURE CITED

1. D. B. Spalding, "Heat transfer for turbulent separated flows," J. Fluid Mech., 27, Pt. 1, 97-109 (1967).
2. V. K. Migai, "Toward a theory of heat transfer in a turbulent flow with separation," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 170-174 (1967).
3. K. M. Krall and E. M. Sparrow, "Turbulent heat exchange in regions of flow separation and attachment and the region of developed flow after reattachment in a circular pipe," Teploperedacha, No. 1, 145-152 (1966).
4. E. G. Filetti and V. M. Case, "Heat exchange in regions of flow separation and attachment and the region of developed flow after a double projection at the inlet of a planar channel," Teploperedacha, No. 2, 51-57 (1967).
5. Zemanik and Dugall, "Local heat exchange after a section of abrupt expansion in a circular pipe," Teploperedacha, 92, No. 1, 54-62 (1970).
6. J. Laufer, "The structure of turbulence in fully developed pipe flow," NASA Rep. 1174, 1-18 (1954).
7. A. A. Zhukauskas, Convective Heat Transfer in Heat Exchangers [in Russian], Nauka, Moscow (1982).
8. A. S. Sukomel, V. I. Velichko, and Yu. G. Abrosimov, Heat Exchange and Friction in the Turbulent Flow of Gas in Short Channels [in Russian], Énergiya, Moscow (1979).